Variations of Landing Distance of Fixed-Wing Aircraft in STOL Operations

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Statistical information on the variability of approach gradient, threshold height, threshold speed, touchdown speed, coefficient of braking friction, time of initiation of the controls, and aerodynamic drag is used as a basis for establishing the variability of parameters that may be expected in STOL operations. Departures from this statistical information were made to reflect the ability of the aircraft to fly slowly and to touchdown early. Because of the nonnormality of the distributions of certain parameters and the nonlinearity of the functions that link the distance S to others, the combination of the distributions is complex. Simplifications are discussed and adopted. The distance SP corresponding to a probability P can be found, with good approximation, when distances traversed with and without reverse thrust are known. An increase of the aerodynamic drag reduces the variability of the distance. The reverse thrust increases such variability. Thus, an assessment of the merits of a new device not based on consideration of S_p may be misleading. It is regarded as important that further studies of the type presented here be undertaken and that more statistics on the variability of the parameters be collected.

Nomenclature†

D_A, D_G = $D_A = D$ (drag) airborne; $D_G = D$ groundborne
E = error in recognizing V_t
E_1, E_2 = $E_1 = E$ at $V_{CO\ min}$; $E_2 = E$ at $V_{CO\ max}$
F = field length factor
M_2 = second moment of a distribution
M = speed margin
M_1, M_2 = $M_1 = M$ between V_{TMD} and $V_{CO\ min}$; $M_2 = M$ between V_{A_t} and $V_{CO\ max}$
P = probability
R = radius of curvature
S = landing distance, or a particular landing distance where subscripts are omitted
S_a, S_g = S_a = airborne part of S; S_g = groundborne part of S
S_P, S_R = $S_P = S$ corresponding to P; S_R, a reference landing distance (see Sec. I)
T = reverse thrust produced by one power unit
V_d = according to context, either speed at which path is tangent to horizontal, touchdown speed, or a speed which is both
V_f, V_t = $V_f = V$ at beginning of flare; V_t = threshold‡ speed
V_{Ad}, V_A = average of V_d or V_t, respectively
V_{d_n} = V_d corresponding to load factor n
$V_{CO\ max}$ = speed such that if V_t exceeds it, the attempt to land is discontinued
$V_{CO\ min}$ = speed such that if V_t is below it, the attempt to land is discontinued
V_{TMD} = minimum demonstrated threshold speed
α = V_d^2/V_t^2
h = height above runway
h_f, h_t = $h_f = h$ at beginning of flare; $h_t = h$ at threshold
n = load factor L/W (L = lift, W = weight)
n_d, n_t = $n_d = n$ at V_d; $n_t = n$ at V_t
$(n_t)_1$ = n_t giving minimum distance
γ = D/W
γ = particular value of γ
θ_r = slope of runway
θ_t = slope of path at threshold
l = $\psi\ V_t^2/V_s^2$
λ_d, λ_t = $\lambda_d = \lambda$ at V_d; $\lambda_t = \lambda$ at V_t
μ = coefficient of braking friction
μ_0 = μ for reference wet runway
ν = $2T/W$
ν_1 = particular value of ν
ρ = average D_G/D_A
σ = standard deviation
σ_x = σ of variable x ($x = V_t$ or μ/μ_0, etc. or S)
$\sigma_{x\ residual}$ = residual σ of variable x correlated to another ($x = V_d$ or h_t etc.)
τ = time at which a control movement is initiated
τ_1, τ_2, τ_3 = $\tau_1 = \tau$ for reverse thrust; $\tau_2 = \tau$ for wheel brakes; $\tau_3 = \tau$ for spoilers
φ = coefficient defined in Sec. VI
ψ = $C_L/C_{L\ max}$
ψ_d, ψ_t = $\psi_d = \psi$ at V_d; $\psi_t = \psi$ at V_t
$(x)_{op}$ = indicates that parameter x ($x = n_t$, or i, or ψ, etc.) corresponds to optimum n_t

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† Other symbols (C_L, $C_{L\ max}$, D, L, V, V_S, W, and g) have their conventional meanings.
‡ The threshold is the approach end of the runway. However, if the approach obstruction clearance rules make the end of the runway unusable, then the intersection of the approach clearance plane with the runway is deemed to be the threshold. In this sense, the definition is similar to that of the statistical sources, Refs. 1–8.

Introduction

THE aim of this paper is to investigate, in respect to landings in STOL operations with fixed-wing aircraft, the relationship between a landing distance and the probability of that distance being exceeded. Relationships of this kind can be used for several purposes. One is to investigate the effect of a device, accounting not only for its direct effect on the landing distance, but also for its effect in modifying the variability of the distance. Another is to enable the manufacturer to advise on the landing distance that should be available to operate the aircraft safely.

Relationships between landing distance and the probability of that distance being exceeded in operation have, up to now, been established mainly for the purpose of proposing or adopting safety requirements. The major part of the work has been done in the United Kingdom, which was the first to propose that International Civil Aviation Organization (ICAO) standardize landing distance requirements on the basis of probability calculations. The proposals submitted were aimed at replacing the present rules for landing distances (1.3 V_S, 50 ft, dry runway, little or no credit for reverse thrust, field length factor 1.67) by rules providing for the measurement of a distance more representative of the average landing

Table 1 Properties of paths

Shape of path	a satisfies	n_d	$h_f \cdot 2g/V_t^2$	λ_t	λ_d
Constant C_L	$n_t(1 - a) + lg\, a = \gamma^2$	$n_t a$	$(\gamma^2/n_t - 1)\chi^a$	n_t	n_t
Constant n	$lg(1/a) = \gamma^2/n_t - 1$	n_t	$\gamma^2/n_t - 1$	n_t	n_t/a
Circle	$(1 - a) = \gamma^2/n_t - 1$	$(n_t - 1)a + 1$	$\gamma^2/n_t - 1$	n_t	$n_t - 1 + 1/a$

[a] No simple relation. An approximation value is $\chi = 1 + 0.178\,[(1 - a)^2/\gamma^2] - 0.076\,[(1 - a)^4/\gamma^4]$.

technique used in operation and for the application to that distance of a field length factor catering for the variations. The statistics on the distribution of the parameters that affect the landing distance were then taken, generally in respect of civil transport aircraft engaged in scheduled services, by most of the countries that participate in the work of the ICAO Airworthiness Committee. No other statistics are available. No attempt seems to have been made as yet to establish "distance-probability" relationships for those classes of aircraft or operations for which there are important differences of technique as compared with the average found in the statistical analysis.

I. Method

For a given set of aircraft characteristics and operational parameters (altitude, average wind, temperature, etc.), the landing distance achieved in operation varies. The distribution of the landing distance can be established by synthesis, using the distributions of the variable parameters. An increment dx of variable x produces an increment of distance $(\partial S/\partial x)dx$, and this permits one to obtain the distribution of the distance S where the parameter x is the source of variation. The distribution of the distance as dependent upon all of the parameters, which are sources of this variation, is then obtained by combining individual distributions by the classical methods described in standard probability texts, such as Ref. 9.

The accuracy of the resultant distribution so obtained depends upon the reliability of the individual distributions of parameters and on the accuracy of the partial derivatives $\partial S/\partial x$. The reliability of the individual distributions, in turn, depends upon the amount of statistical data collected and upon the manner in which they have been analyzed and interpreted. Although considerable efforts have been made in this respect,[1–8] it is highly desirable that more data be collected, particularly in respect of the parameters found, in studies like the present one, to have the dominant contribution. It is also highly desirable that more analysis of the existing data be made and published.

As to the determination of the partial derivatives, in principle they should be derived from tests. Since practically this would be too expensive, a reasonable degree of accuracy can be reached in establishing, by calculations, a set of values of the distance, here called the "reference distances" and denoted S_R, as well as the derivatives. If, in the calculations, the adopted approximations affect the values of the derivatives and the reference distances proportionally, S_P may be taken as proportional to the measured value of S_R.

II. Aircraft Envisaged

There is, at present, no standard definition of the STOL operation. Certain authors[10] would make the "use of the propeller slipstream or other forms of propulsion to augment the wing lift" a prerequisite. Others[11] would require that the takeoff and landing be made at speeds below the power-off stalling speed. It is, however, not certain that, if the comparison of the distances achieved by various aircraft were made on the basis of a common level of safety, the advantage of any design feature would be as great as where the comparisons are made on the basis of the minimum distance achievable. Until this question is resolved, the STOL appellation should not be denied to any aircraft merely because of the absence of any particular feature.

A first step in the resolution of the problem is to establish the distribution of the landing distance in STOL operations, conceived as operations that differ from the average operations (observed where the statistical information was obtained), in the following respects. The operation is undertaken at a remote place where the building of a long runway is impractical and where the removal of obstructions should be restricted to the bare minimum. The aircraft used is able to fly slowly, has a high drag in the airborne portion of the landing, and is equipped with strong retardation means; its design features and handling characteristics are such that a relatively high precision can be achieved. The pilot will achieve the relatively high precision that is consistent with the qualities of the aircraft, where the runway is critical. A second step, not made in this study, consists in introducing new aircraft characteristics and new sources of variation, in order to deal, inter alia, with the case of lift augmentation or any other design features.

III. Derivatives

Airborne Path

Three forms of path are envisaged, all of them tangent to the runway: 1) a circular path defined by $R = V^2/g(n - 1) = V_t^2/g(n_t - 1)$, 2) a path defined by $n = n_t$ or "constant n" path, and 3) a path defined by $n = (n_t/V_t^2)V^2$ or "constant C_L" path. Since the power does not vary appreciably during the airborne path, γ is considered as constant, giving the properties in Table 1.

Optimum Values

In making $d\lambda^{35}{}_d/dn_t = 0$ (circular and constant n) or $dn_t/da = 0$ (constant C_L path), an optimum value of n_t is found as the value, which provides the largest margin between C_L and $C_{L\,\max}$. The optimum values are indicated in Table 2. The values in Table 2 are referred to as $(n_t)_{op}$, $(a)_{op}$, $(\lambda_d)_{op}$, etc. The differences between the three paths are large only where γ is large and n_t small. The curvature of the constant n path increases from V_t to V_d. The curvature of the constant C_L path decreases from V_t to V_d, and where $n_t = (n_t)_{op}$, the radius of curvature is infinite at V_d.

Table 2 Optimum values

Shape of path	n_t satisfies	a	i_d	n_d
Constant C_L	$\gamma^2 + lgn_t = n_t - 1$	$1/n_t$	n_t	1
Constant n	$n_t = 1 + \gamma + \gamma^2/2$	$\exp(-2\gamma/2 + \gamma)$	$(1 + \gamma + \gamma^2/2)\exp(2\gamma/2 + \gamma)$	$1 + \gamma + \gamma^2/2$
Circle	$n_t = 1 + \gamma + \gamma^2$	$1/1 + \gamma$	$(1 + \gamma)^2$	$1 + \gamma$

Minimum Threshold Speed

For a constant C_L path, it can be shown that, at $(n_t)_{op}$, $V_t^2 = V_s^2(l_d)_{op}/\psi_d$, so that $V_t^2 = V_s^2(\lambda_d)_{op}$ represents a theoretical minimum threshold speed. At a lower V_t, it is not possible to have a path tangent to the runway.

Minimum Distance for a Value of ψ

It can be shown that the optimum value of n_t is not the value giving the minimum distance for a value of ψ. A distance that is shorter can be obtained at a threshold speed slightly greater than that corresponding to the optimum, using $(n_t)_1 > (n_t)_{op}$.

Selection of Average Path

No one of the three paths envisaged represents closely the paths used in operation. The pitching inertia will prevent the instantaneous establishment of n_t. Moreover, following one of these three paths would require instantaneous information and instantaneous response. Any typical path should however represent an average of all of the paths that start at approximately the same glide angle with approximately the same V_t and finish at approximately the same V_d. It is believed that the substitution will affect proportionally S and $\partial S/\partial x_i$.

Where the problem is to determine an ultimate property, like the theoretical minimum threshold speed, the constant C_L path should be used. For conditions other than the extremes, the constant C_L path taken at $(n_t)_{op}$ has the disadvantage of having at V_d an infinite radius; in the last part of the flare, the aircraft is already near the ground, and if the lift can be reduced (by spoilers, for instance), it can touch down and shorten the distance. Such a path does not represent the average conditions near touchdown in STOL operations.

For a normal STOL operation, the proportion of the lift available, used for flaring, is increasing from V_t to V_d since the consequences of a stall are less serious near the touchdown point. Both the constant n and the circular paths satisfy this condition. It must be assumed that the application of a load factor $n_t < (n_t)_{op}$ that gives $\psi_d > (\psi_d)_{op}$ and $S > (S)_{op}$ will be avoided as much as possible. The tendency will be to use $(n_t)_{op}$ rather than $(n_t)_1$, so that whatever V_t is, the pilot handles the aircraft in the manner, which provides for the largest margin between C_L used for flaring and $C_{L\,max}$. The constant n and the circular path are not very different, so that the circular path can be considered as an acceptable approximation of the constant n path that does not affect $[(\partial S/\partial x_i)/S]$ appreciably. Therefore, it is assumed that the circular path, taken with $n_t = (n_t)_{op}$ of Table 2, is representative of the average path.

Expressions of Derivatives

The statistical information (Sec. IV) indicates that h_t and V_d are partially correlated to V_t. For STOL aircraft, γ is partially correlated to V_t also. For a circular arc, $(n_t)_{op}$,

$$\frac{dV_d}{dV_t} = \frac{1}{(1+\gamma)^{1/2}}\left(1 - \frac{V_t}{2(1+\gamma)}\cdot\frac{d\gamma}{dV_t}\right) \tag{1}$$

This gives

$$\frac{dS}{dV_t} = \frac{V_t}{g(1+\gamma)} + \frac{1}{\gamma}\frac{dh_t}{dV_t} - \left(\frac{h_t}{\gamma^2} + \frac{V_t^2}{2g(1+\gamma)^2}\right)\frac{d\gamma}{dV_t} + \frac{\partial Sg}{\partial V_d}\cdot\frac{dV_d}{dV_t} + \frac{dSg}{\partial\gamma}\cdot\frac{d\gamma}{dV_t} \tag{2}$$

For the residual variations of h_t, V_d, and γ,

$$\frac{\partial S}{\partial h_t} = \frac{1}{\gamma} \qquad \frac{\partial S}{\partial V_d} = -\frac{V_d}{g\gamma} + \frac{\partial S_g}{\partial V_d}$$
$$\frac{\partial S}{\partial \gamma} = -\frac{h_t}{\gamma^2} - \frac{V_t^2}{2g(1+\gamma)^2} + \frac{\partial Sg}{\partial\gamma} \tag{3}$$

For the parameters that do not affect the airborne distance,

$$\partial S/\partial(\mu/\mu_0) = \partial S_g/(\mu/\mu_0) \qquad \partial S/\partial\tau = \partial S_g/\partial\tau \tag{4}$$

IV. Summary of Statistical Information Available

The statistical information available[1–8] is relative to operations of civil aircraft, generally in scheduled services, and to landings at large airports where the threshold is the end of the runway or the intersection of the runway with the approach clearance plane. It can be summarized as in Table 3. In this table the subscript A represents the average. Values are conservative for critical runways.

Threshold Speed

The average is 1.3 V_{S0} for aircraft having V_{S0} <70 knots, V_{S0} being relative to the landing configuration at maximum landing weight. The threshold speed of faster aircraft depends upon the stalling speed in the approach configuration V_{S1}. The standard deviation, 5 knots, is independent of the aircraft characteristics because of the great sensitivity of slow aircraft to turbulence; however certain data[8] show a slight reduction of σ_{Vt} with V_{At}. The cutoffs, indicated as being at $V_{At} \pm 15$ knots, reflect the fact that landings at $V_t > V_{At} + 15$ knots or $V_t < V_{At} - 15$ knots are less frequent than the normal law would indicate. Pilots tend to discontinue the attempt to land if the speed is too high or too low and if they cannot correct it. The value 15 knots or $3\sigma_{Vt}$ would confirm the rate of balked landings. σ_{Vt} does not seem to be affected by head winds, cross winds, or visibility conditions, except for the very drastic ones.

Threshold Height

The distribution is such that large heights are less frequent, and heights slightly under the average are more frequent than the normal law would indicate. The height and its variations seem to depend more upon local conditions than upon the type of aircraft.

Touchdown Speed

The relation between V_d and V_t and the regression dV_d/dV_t seem to be influenced strongly by the aircraft characteristics, but turbulence, wind, and runway criticality do not seem to affect V_d more than is indicated by its correlation to V_t.

Braking Friction

In the present study, the landing is assumed to take place on a wet runway, because this corresponds to conditions frequently encountered. In a study of the effect of a device, it may also be useful to assume a runway covered by slush and/or snow. The methods to be used would be the same as those described in this study. The relation (coefficient of braking friction vs speed) defined as the reference wet runway by the ICAO Standing Committee on Performance (SCP) (1953) has received good confirmation since, namely, in NASA research.[12] The standard deviation of 13% was also established by the SCP. It represents 11% variation in brake pressure, torque-limiting devices, and brake control mechanisms, and 6% variation between runway surfaces.

Aerodynamic Drag

The statistics indicate the variation of the basic drag or variation from aircraft to aircraft of the same series. Of course, for a given aircraft, the drag will be correlated to the threshold speed.

V. Selection of Distributions for STOL Landing

The differences between the operations observed where the statistical information was obtained and the STOL opera-

Table 3 Summary of statistical information[a]

Parameter	Symbol	Average	Distribution	σ or $\bar{M}e_2^{1/2}$
Gradient of approach	θ_t	$\theta_{t,A} = 4\%$	Normal	$0.3\ \theta_{t,A}$
Threshold speed	V_t	$V_{A_t} = V_{S1} + 24$ knots, or $V_{A_t} = 1.3\ V_{s0}$	Normal, with cutoffs at $\pm$ 15 knots	$\sigma_{V_t} = 5$ knots
Threshold height	h_t	$h_{A_t} = 20$ ft	Not normal, correlated to V_t	Residual $\bar{M}_2^{1/2} = 11$ ft, $(dh_t/dV_t) = 0.27$
Touchdown speed	V_d	$V_{A_d} = V_{A_t} - 15$ knots	Normal, correlated to V_t	Residual $\sigma_{V_d} = 3.5$ knots, $(dV_d/dV_t) = 0.6$
Braking friction	μ	μ_0 reference, wet surface[a]	μ/μ_0 Normal	$\sigma(\mu/\mu_r) = 0.13$
Initiation, reverse thrust	τ_1	3 sec after touchdown	Normal	$\sigma_{\tau 1} = 1.5$ sec
Initiation, other controls	τ_2 τ_3	...	Normal	$\sigma_{\tau 2} = \sigma_{\tau 3} = 1$ sec
Basic aerodynamic drag	D_A	...	Normal	$\sigma_D = 0.022\ D_A$

[a] From 0 to 87 knots $1/\mu_0 = 1.28 + 1.80\ V^{1.08}$, where V is in hundred feet per second.

tions as dealt with in this study are summarized in Sec. II. The trends of variations of the parameters, as recorded in Sec. IV, do not seem to be affected by those differences. However, some of the numerical values of the characteristics of the distributions, also indicated in Sec. IV, may depend upon the aircraft characteristics or the average landing technique. Where it appears evident that those numerical values are not applicable to STOL operations, other numerical values will be established; otherwise the numerical values in Sec. IV will be adopted. The selected values are grouped in Table 4.

Gradient of Approach

The total energy, kinetic plus potential, of the aircraft is equal to the work of the drag so that to obtain a short landing distance it is essential that all of the drag available at every moment should be used. Therefore, the path in the last portion of the approach should be as steep as possible. A second reason for a steep approach is the presence of obstructions. It is assumed that the slope at the approach is $\theta_t = \gamma$. The variability of the landing distance caused by the variability of the gradient of approach near the slope envisaged is small, and this effect will be ignored.

Threshold Height

Because of the presence of obstructions, threshold heights of the order of 20 ft are insufficient. It is assumed that the threshold height is 50 ft where the threshold speed is equal to the average V_{At}. The dependence of the height upon the speed is established by the statistics. It may be a result of the fact that estimation of the position of the threshold relative to the aircraft is more difficult when the speed is high. In the absence of better information, the regression $\partial h_t/\partial V_t = 0.27$ will be adopted. As to the residual variation of the height, it can be argued that the error in estimating a distance is proportional to the distance itself, so that the square root of the second moment of the residual distribution of h_t should be 11 ft multiplied by 50/20. However, the distance estimated is not necessarily the height above the runway; the obstructions may help in evaluating the height. Again in the absence of better information, the residual variation characterized by $M_2^{1/2} = 11$ ft will be adopted. For the combination of the distributions, the distribution will be replaced by a normal distribution, $\sigma_{h_t} = 11$ ft.

Threshold Speed

The variations of the threshold speed are caused by 1) the difficulty of estimating the height (which may be converted to speed), estimating the speed itself, instrument errors, and errors in reading the instrument; and 2) the need to increase the threshold speed in turbulent air. Cause 2 is the more important of the two. This is borne out by the statistics. In calm air, all or almost all of the lift available can be used to achieve the flare. A greater margin between the C_L used for flaring and $C_{L\max}$ is needed in turbulent atmosphere, and this greater margin is obtained by an increase in V_t.

The statistics show the existence of a cutoff speed $V_{CO\max} = V_{At} + 15$ knots. Since V_{At} is defined as the average of a symmetrical distribution, there must be a second cutoff speed at $V_{CO\min} = V_{At} - 15$ knots. For a STOL aircraft having $V_S = 59$ knots, taking $V_{At} = 1.3\ V_S$, $V_{CO\min} = 61.8$ knots or $1.048\ V_S$. This speed would permit a flare using $C_L = C_{L\max}$ all of the time during the flare, only if $\gamma \leq 0.08$. STOL aircraft have a ratio γ greater than that.

Table 4 Assumptions concerning variability of parameters

Parameter	Symbol	Average	Distribution	σ and regression
Gradient of approach	θ_t	$\theta_t = \gamma$	Normal	Neglected
Threshold height	h_t	$h_{At} = 50$ ft	Normal, correlated to V_t	$\sigma h_t = 11$ ft; $dh_t/dV_t = 0.27$
Threshold speed	V_t	$V_{At} = V_{TMD} + M_1 + 8$ knots	Normal, cutoffs at $V_{At} \pm 8$ knots	$\sigma_{Vt} = 4$ knots
Touchdown speed	V_d	"a" in Table 2; $V_{Ad} = V_{At}(a)^{1/2}$	Normal, correlated to V_t	Residual: Table 5; dV_d/dV_t, Eq. (1)
Braking friction	μ	μ_r = reference wet surface	μ/μ_0, normal	$\sigma\mu/\mu_0 = 13\%$
Aerodynamic drag	γ	Aircraft characteristics	Normal, correlated to V_t	Residual: 2.2%; $\frac{d\gamma}{dV_t} = 1.2\left(\frac{\gamma}{V_t}\right)$
Recognition of threshold speed	E_1 E_2	$E_1 = 0$ at $V_{CO\min}$; $E_2 = 0$ at $V_{CO\max}$	Normal; Normal	$\sigma_{E1} = 2$ knots; $\sigma_{E2} = 3$ knots
Initiation of reverse thrust, brakes, spoilers	τ_1 τ_2 τ_3	1.5 sec after t.d.; 0 sec after t.d.; 0 sec after t.d.	Normal, correlated together	$\sigma_\tau = 1$ sec

This shows that the information in the statistics concerning the threshold speed and its variations is not directly applicable.

As already indicated, there is a theoretical minimum threshold speed compatible with the zero vertical speed at impact. Practically, a smaller speed may be possible, depending upon the landing gear characteristics, or a larger speed may be needed, depending upon the handling characteristics at low speed. Therefore, it is logical to measure a speed V_{TMD}, defined as the minimum speed obtained at 50 ft above the landing surface, at and above which it is demonstrated that the airplane can be made constantly to complete an approach, touchdown and landing, without displaying any hazardous characteristics. This speed will serve as the starting point to determine other threshold speeds.

It must be expected that pilots will not approach at speeds as low as V_{TMD} in operation. The minimum threshold speed will be $V_{TMD} + M_1$, where the margin M_1 should provide for error in estimating the speed or in taking the decision to abandon the attempt to land. In order to keep V_t as low as possible, the pilot should know the speed at which the attempt to land should be discontinued. The minimum cutoff speed, equal to $V_{CO\ \min} = V_{TMD} + M_1$, is thus a piece of information given to the pilot, together with an explanation of its significance. M_1 should be large enough to insure that the probability of a landing started at $V_t \leq V_{TMD}$ is below a selected value P_1. The margin M_1 need not account for turbulence, because a landing near V_{TMD} would not be attempted in turbulent atmosphere. Thus $M_1 = M_1(P_1, E_1)$ where E_1 is the error in recognizing the threshold speed near V_{TMD}, introduced as a new parameter affecting the landing distance.

Defining M_2 as the margin between $V_{CO\ \min}$ and V_{A_t}, and assuming, as in the statistics, that V_{A_t} is the average of a symmetrical distribution of V_t,

$$V_{A_t} = V_{CO\ \min} + M_2 \qquad V_{CO\ \max} = V_{A_t} + M_2$$

This would mean that the attempt to land would be discontinued if V_t cannot be reduced to a value equal to or lower than $V_{CO\ \max}$. Ideally M_2 should be large enough to insure that the probability of a balked landing is smaller than a selected value P_2, of the order of $\frac{1}{800}$ recognized by the statistics, and M_2 would be easily established if σ_{V_t} were known.

Practically, the problem may be somewhat more complex. The statistics indicate that the pilots abandon their attempts to land where V_t is greater than $V_{A_t} + 3\sigma_{V_t}$, but a large majority of the aircraft observed had been certificated on the basis of requirements (1.3 V_S, gliding approach, 50 ft dry hard concrete and little or no credit for reverse thrust, field length factor 1.67), that did not specify the speed at which the attempt to land should be abandoned. It may thus be argued that the statistics reflect the fact that the pilots discovered, by trial and error, that, when their speeds were 15 knots above the average, the distance available was insufficient. It is thus quite possible that, in giving to the pilots a clear indication of the speeds at which the attempt to land should be abandoned, and making $M_2 < 15$ knots, more precision (a reduction of σ_{V_t}), rather than an increase in the rate of balked landings, should be expected.

The confirmation of $M_2 = 3\sigma_{V_t}$ by the rate of balked landings cannot be accepted without some reservation. A speed $V_t > V_{A_t} + 3\sigma_{V_t}$ is an incentive to abandon the attempt to land only if the runway is critical. On the other hand, excessive speed may be one of the causes for abandoning the attempt, but it is not the only one. The distribution of the threshold speed may be such that the frequency of speeds below $V_{A_t} + 3\sigma_{A_t}$ is higher than the frequency corresponding strictly to the normal distribution, and speeds slightly above $V_{A_t} + 3\sigma_{A_t}$ may be less frequent. This would reflect the tendency to correct the speed where it is high. In combining the distribution with others, to obtain the distribution of S, the effect would be immaterial; but in estimating the balked landing rate, a considerable error would be made. In the foregoing, the theoretical rate of balked landings, obtained as if the runway were critical and the distribution strictly normal, is given as an indication rather than an absolute value.

For a specific example of STOL aircraft, with $V_S = 59$ knots, V_{TMD} would be estimated, consistent with experience, to be 65.6 knots; M_1, corresponding to $P_1 = 10^{-4}$ and $\sigma_{E_1} = 2$ knots, would be 3 knots, making $V_{CO\ \min} = 68.6$ knots or 1.16 V_S. If $V_{A_t} = 1.3\ V_S$ (applying the figure given in the statistics), V_{A_t} would be 76.6 knots, resulting in $M_2 = 8$ knots. Some data in Ref. 8 show a tendency for σ_{V_t} to be smaller than 5 knots for slow aircraft. If $\sigma_{V_t} = 5$ knots is valid for $V_S = 100$ knots, a proportion of σ_{V_t} to V_S would make $\sigma_{V_t} = 3$ knots, but the simple proportionality of σ_{V_t} to V_S is not substantiated. In taking $M_2 = 8$ knots, this should be considered as $2\sigma_{V_t}$ or $\sigma_{V_t} = 4$ knots.

The figures $V_{A_t} = 1.3\ V_S$ and $\sigma_{V_t} = 4$ knots appear rather conservative, since in tests of STOL aircraft V_{A_t} was found to be 1.2 V_S and $\sigma_{V_t} = 3.7$ knots. No threshold speed out of the range $V_{A_t} \pm 2\sigma_{V_t}$ was found. For the continuation of this study, σ_{V_t} will be taken as 4 knots and $M_2 = 2\sigma_{V_t}$ (or 8 knots).

Touchdown Speed

The statistics show that $V_{A_d} = V_{A_t} - 15$ knots, and $dV_d/dV_t = 0.6$. Taking again the example of an aircraft having $V_S = 59$ knots, $\sigma_{V_t} = 4$ knots, and $M_2 = 2\sigma_{V_t}$, at $V_t = V_{A_t} = 76.6$ knots, $V_d = 61.6$; at $V_t = V_{CO\ \min} = 68.6$ knots, $V_d = 56.8$; and at $V_t = V_{CO\ \max} = 84.6$ knots, $V_d = 66.4$. Taking (σ_{Vd}) residual as in the statistics, 3.5 knots, the probability of touching down at $V_d < V_S$, where $V_t = V_{CO\ \min}$ is three in four. Taking the resultant standard deviation (combination of correlated and residual), which is 4.24 knots, it can be shown that the probability of a touchdown at $V_d < V_S$, considering that all of the landings between $V_{CO\ \min}$ and $V_{CO\ \max}$, is 0.27 or more than one in four. The proportion of landings at $V_d < V_S$ obtained in applying the statistical information to STOL operations is too high to be representative. To obtain a better representation, it will be considered first that at each value of V_t one-half of the landings follow a path more incurvated than the average path and one-half of a path less incurvated. At the small values of V_t, say at $V_{CO\ \min}$, the less incurvated path terminates at $V_d = V_S$. At the average value of V_t (denoted V_{A_t}), the more incurvated path is the one for which all of the lift available is used for flaring, as if the turbulence had not required V_t to be as high as V_{A_t}. It is then assumed that the load factor n corresponding to this path is not exceeded even when V_t becomes as high as $V_{CO\ \max}$.

Thus a first view of the distribution is that it is a rectangular one. Numerical examples show that, at the three values of V_t (minimum, average, and maximum), the limits of the rectangular distributions so obtained are about the same. The distribution of S due to V_d, residual, as the source of variation, is established in view of its combination with the distributions due to all of the other sources; but since many distributions are involved, by virtue of the central limit theorem, the resultant tends to be normal, and the contribution of each individual one is equivalent to that of a normal distribution having the same second moment. A second view of the distribution is thus that it is a normal distribution having the same second moment as the rectangular distribution having the limits indicated previously.

Those two views are illustrated in Table 5. The first column indicates the three values of the threshold speed, and the second one indicates the designation of the correlated value of V_d. The probabilities P (5th and 6th columns) are those that V_d is over the range defined by the lower limit (3rd column) and the upper limit (4th column).

The last two columns of the table give an indication of the range of the proportion of the lift used for flaring (either 1, or "op" if the optimum ratio is assumed), together with an indication as to whether there is a floating period ("fl" mentioned in those cases). It should be observed, however, that where the indication is "1, fl" this is only an indication of the general curvature of the path; the path may be less curved than that corresponding to $C_L/C_{L\,\max} = 1$, with a positive sinking speed at impact; but then the floating period becomes different. Also, the indication $C_L/C_{L\,\max} > 1$ does not necessarily mean that a path is impossible to achieve but only that the sinking speed at impact is positive.

The distribution resulting from the foregoing considerations is a conservative one. This is ascertained by comparing it, in numerical examples, with the distribution of V_d in the statistics. The resultant distribution of V_d in the statistics has a standard deviation of 4.6 knots. In the basic numerical example in the following section, the resultant is 4.20 knots, although V_S is of the order of 60% of V_S corresponding to the statistics.

Braking Friction

Since this parameter does not depend on the type of aircraft or mode of operation, the braking friction is taken as in Table 3.

Aerodynamic Drag

The aerodynamic drag is a function of the threshold speed. The dependence is found in the polar. For this study, it is estimated that the aerodynamic drag is partially correlated to V_t, the regression being $d\gamma/dV_t = 1.2(\gamma/V_t)$. It is assumed that there is a residual variation of the aerodynamic drag representing the difference in drag between air frames of the same series in the landing configuration, and also the differences in drag for a given air frame, caused by wearing of parts, small repairs, etc. The figure in Table 3 is adopted.

Recognition of Threshold Speed

There is no statistical information on the distribution of the error made in estimating the threshold speed. Since the aircraft passes the threshold in steady flight, the error may be somewhat smaller than the 3.5 knots assumed in other studies,[2] and $\sigma_E = 3$ knots is adopted here. To establish M_1, however, a somewhat more optimistic value of 2 knots may be adopted because an attempt to land at a threshold speed close to the minimum will be made in calm air and good conditions of visibility; the fact that the landing may have to be abandoned for insufficient speed may be the primary preoccupation of the crew.

Time at Which a Control Movement Is Initiated

It is envisaged that three control movements are initiated after the aircraft passes the threshold: those of the spoilers, the wheel brakes, and the reverse thrust. The spoilers are assumed to be operative at touchdown. A delay in the operation of the spoiler control is one of the causes for introduction of a floating period. It is accounted for as a variation of the touchdown speed. The spoiler action that results in increasing the load on the wheels, the action of the wheel brakes, and the action of the reverse thrust will be discussed together for the following reasons: 1) the information summarized in Table 3 is not entirely satisfactory, since it is difficult to accept that the standard deviation of the time of application of the reverse thrust should be 1.5 times that of the other control movements; and 2) logically there is some correlation between the three times of application, since the cause for a long delay on one will generally affect the three. In fact, for small values of $\tau_1, \partial S/\partial\tau_1$ is much greater than $\partial S/\partial\tau_2$ and $\partial S/\partial\tau_3$; for large values of τ_1, $\partial S/\partial\tau_1$ tends to become smaller, but $\partial S/\partial\tau_2$ and $\partial S/\partial\tau_3$ become larger. Since the cause of a large τ would also be a cause of large τ_2 and τ_3, the three delays can then be considered as correlated. Numerically this gives a constant value of the combined effect of the three delays, and a unique delay τ is considered. It is not certain that a constant figure of 1.5 sec is adequate for a range of aircraft that includes STOL. σ_τ may be proportional to the time taken to stop the aircraft without reverse thrust, and, assuming that 1.5 sec corresponds to an aircraft having the average characteristics of the aircraft observed, σ_τ should be of the order of 0.6 sec. It is thus conservative to take $\sigma_\tau = 1$ sec.

VI. Parametric Study

Definition of φ

The retarding force in the groundborne part of the landing can be expressed as

$$D_G = D_A \frac{V'^2}{V_d'^2} + \eta\mu\left(1 - \beta \frac{V'^2}{V_d'^2}\right) W + \theta_r W + 2T$$

(two-engine aircraft assumed), where V' and $V_d'^2$ are air speeds. For a zero wind condition with $\eta = 1$, $\beta = 1$, and

Table 5 Probability of ranges of V_d

V_t	Designation of V_d correlated to V_t	Limits of range V_d: Lower	Upper	P, assuming distribution: Rectang., %	Normal, %	$C_L/C_{L\,\max}$: Min	Max
$V_{CO\,\min}$	$V_{d\,\min}$	0	$V_S - \frac{1}{2}(V_{d\,\min} - V_S)$	0	0.47	1, fl	1, fl
		$V_S - \frac{1}{2}(V_{d\,\min} - V_S)$	V_S	0	3.71	1, fl	1, fl
		V_S	$V_{d\,\min}$	50	45.8	1, fl	1
		$V_{d\,\min}$	$V_{d\,\min} + (V_{d\,\min} - V_S)$	50	45.8	1	...
		$V_{d\,\min} + (V_{d\,\min} - V_S)$	$V_{d\,\min} + \frac{3}{2}(V_{d\,\min} - V_S)$	0	3.71	>1	...
		$V_{d\,\min} + \frac{3}{2}(V_{d\,\min} - V_S)$	∞	0	0.47	>1	...
V_{At}	V_{Ad}	0	$V_{Ad} - \frac{3}{2}(V_{dn} - V_{Ad})$	0	0.47	op fl	op fl
		$V_{Ad} - \frac{3}{2}(V_{dn} - V_{Ad})$	$V_{Ad} - (V_{dn} - V_{Ad})$	0	3.71	op fl	op fl
		$V_{Ad} - (V_{dn} - V_{Ad})$	V_{Ad}	50	45.8	op fl	op
		V_{Ad}	V_{dn}	50	45.8	op	1
		V_{dn}	$V_{dn} + \frac{1}{2}(V_{dn} - V_{Ad})$	0	3.71	1	>1
		$V_{dn} + \frac{1}{2}(V_{dn} - V_{Ad})$	∞	0	0.47	>1	...
$V_{CO\,\max}$	$V_{d\,\max}$	0	$V_{d\,\max} - \frac{3}{2}(V_{dn} - V_{d\,\max})$	0	0.47	op fl	op fl
		$V_{d\,\max} - \frac{3}{2}(V_{dn} - V_{d\,\max})$	$V_{d\,\max} - (V_{dn} - V_{d\,\max})$	0	3.71	op fl	op fl
		$V_{d\,\max} - (V_{dn} - V_{d\,\max})$	$V_{d\,\max}$	50	45.8	op fl	op
		$V_{d\,\max}$	V_{dn}	50	45.8	op	<1
		V_{dn}	$V_{dn} + \frac{1}{2}(V_{dn} - V_{d\,\max})$	0	3.71	<1	→1
		$V_{dn} + \frac{1}{2}(V_{dn} - V_{d\,\max})$	∞	0	0.47	→1	→1

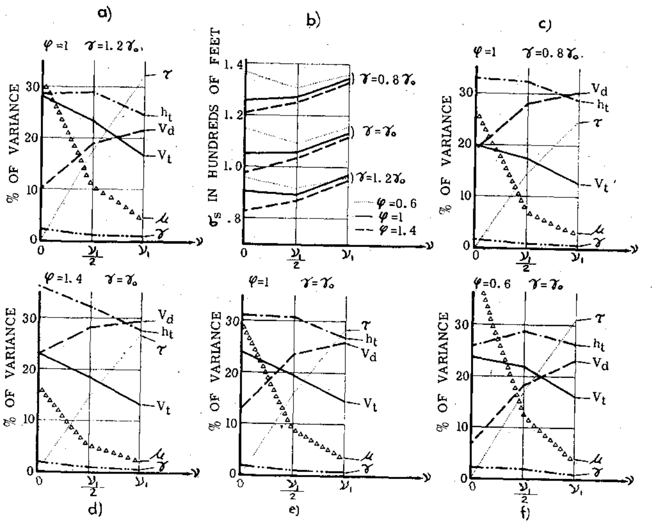

Fig. 1 Standard deviation and analysis of variance; a, c–f) analysis of variance, b) standard deviation.

$\theta_r = 0$, D_G takes the value

$$D_G = D_A(V^2/V_d^2) + \mu[1 - (V^2/V_d^2)]W + 2T$$

The factor φ is defined as the value that satisfies

$$\int_0^{V_d} \frac{VdV}{g\left[\gamma\frac{V'^2}{V_d'^2} + \eta\mu\left(1-\beta\frac{V'^2}{V_d'^2}\right) + \theta_r + \nu\right]} = \int_0^{V_d}\frac{VdV}{g\left[\gamma\frac{V^2}{V_d^2}+\varphi\mu\left(1-\frac{V^2}{V_d^2}\right)+\nu\right]}$$

In this manner, $\varphi = 1$ represents the case where there is no wind, $\eta = 1$ (high efficiency of the wheel braking system), $\beta = 1$ (no spoiler effect), and $\theta_r = 0$. $\varphi < 1$ in cases where η is small, β is large, θ_r has negative values, and the wind is in the direction of landing. $\varphi > 1$ in cases where η is large, β is small, θ_r has positive values, and the wind is opposite to the direction of landing. There is no need to establish explicitly $\varphi = \varphi$ (wind component, η, β, θ_r).

Ranges of Aircraft Characteristics and Operational Parameters

A twin engine aircraft is assumed, having $V_S = 59$ knots. On the average, the wheel brakes and the spoilers are assumed to be effective at touchdown, and the reverse thrust is 1.5 sec after touchdown. The following values are taken: $\gamma_0 = 0.17$ at $V_t = 1.2\ V_S$, sea level, ISA conditions, $\nu_1 = 0.123 + 0.127\ V'$ (V' in hundreds of fps), and $P_1 = 10^{-4}$. (It is estimated that starting a landing at $V_t < V_{TMD}$ is not an incident of great gravity.) This makes $M_1 = 3$ knots. V_{TMD} = the theoretical minimum threshold speed, provided that a circular path does not produce a sinking speed at impact greater than 0.6 fps. σ_{Vd}, residual = 2.58 knots, in conformity with Sec. V. Ranges of aircraft characteristics and operational parameters are explored, making all of the combinations of γ from 0.8 γ_0 to 1.2 γ_0, φ from 0.6 to 1.4, and ν from 0 to ν_1.

VII. Effects of Nonnormality of Distributions

The distribution of S tends to be normal, since it is a resultant of several distributions. The two main causes susceptible of making the distribution of S different from a normal one are 1) the distribution of V_t, account being taken of the cutoffs and of the error in recognizing that V_t is not normal (the effect of the cutoff at $V_{CO\ \max}$ is to render the frequency of large S less than in a normal distribution, and large values of S are critical), and 2) that, although μ/μ_0 is assumed to be distributed normally, the distribution of S due to the variation of μ/μ_0, is skew, because $\partial S/\partial(\mu/\mu_0)$ is not constant. In order to appreciate causes 1 and 2, the distribution of S has been calculated for the three sources of variation, V_t, E_2, and μ/μ_0, by the standard procedure. The standard procedure was also used to combine this resultant with the resultant of the distribution of S for the two sources V_d and τ, although taking $\sigma_t = 1.5$ and neglecting σ_{ht}, residual. On the other hand, another calculation was made replacing the distribution of V_t by a normal one, $\sigma_{V_t} = 4$ knots (thus neglecting the cutoff effects), and taking $\partial S/\partial(\mu/\mu_0)$ at $\mu = \mu_0$. The results of the comparison are shown in Table 6. The simplified procedure permits determining that the order of the contribution of μ to the total variance is 3%, although the contribution of V_t is 17.3%. It can be concluded that the increase of 1.6% of S at $P = 10^{-5}$ obtained by the simple procedure is caused by the fact that the cutoffs are neglected. At $P = 10^{-5}$, the effect of the cutoffs is equivalent to a negative contribution of the order of 8% of the variance in this example, but when the contribution of V_t is large, the simple procedure may introduce appreciable errors.

In the simplified procedure, all of the derivatives are approximated as constants, and this may introduce errors. An increase of V_t from V_{At} to $V_{CO\ \max}$ increases $|\partial S/\partial\tau|$ and $|\partial S/\partial(\mu/\mu_0)|$ and reduces $|\partial S/\partial V_t|$, $|\partial S/\partial V_d|$, $|\partial S/\partial h_t|$, and $|\partial S/\partial\gamma|$. Calculations have been made to determine whether $\partial S/\partial x_i$ should be taken at V_{At} or $V_{CO\ \max}$. It was found that at $\gamma = \gamma_0$, the difference varied from 1 to 4.8%, being greater at $\varphi = 1.4$ than at $\varphi = 0.6$, and that σ_S was, in all of the cases investigated, smaller at $V_{CO\ \max}$ than at V_{At}. Therefore the value of σ_S, established in taking the values of $\partial S/\partial x_i$ at V_{At}, is adopted. In this manner, if an error is introduced, it is a conservative one.

VIII. Distribution of Distance

Standard Deviation

Figure 1b shows σ_S calculated for the 27 combinations γ, φ, and ν envisaged. The subscript "residual," which applies to h_t, V_d, and γ is omitted. Figure 1b shows that 1) without reverse thrust, σ_s is strongly influenced by φ (the aircraft having a poor braking system, or a weak spoiler action, or landing on a downward slope have a large σ_s); 2) a moderate reverse thrust ($\nu = \nu_1/2$) reduces σ_S where φ is low, but it increases σ_S where φ is large; and 3) a strong reverse thrust increases σ_S in all of the cases.

Figure 1e shows the percentage contribution to the variance of the six parameters V_t, V_d, τ, μ, h_t, and γ at $\varphi = 1$ and $\gamma = \gamma_0$. Figures 1d and 1f show the effect of varying φ, and Figs. 1a and 1c show the effect of varying γ.

As indicated previously and confirmed by examples not related here, the combined contributions of the cutoffs and of the error in recognizing V_t would have, for $P = 10^{-5}$, a negative contribution of the order of one-half of the contribution of V_t. Since this is an evaluation rather than a precise figure, it is not traced in Fig. 1. Since at $\nu = 0$, the contribution of V_t is generally of the order of 20 to 25%, all of the σ_S's shown in Fig. 1b for $\nu = 0$ should be reduced by about 5% where $P = 10^{-5}$ is considered. At $\nu = \nu_1$, the

Table 6 Effect of nonnormality

Probability	Standard procedure (S_p, hundreds of feet)	All normal distributions (S_p, hundreds of feet)	Difference, %
10^{-3}	13.75	14.00	1.8
10^{-4}	14.55	14.80	2.5
10^{-5}	15.25	15.50	1.6

contribution of V_t is of the order of 15%, and the correction should be about 3.5%.

Some conclusions can be drawn from an examination of Fig. 1: 1) without reverse thrust, with $\gamma = 1.2\ \gamma_0$ or $\varphi = 0.6$, the largest contribution is that of μ, but a reduction of γ or an increase of φ makes the contribution of $h_{t\ \text{residual}}$ the dominant one; 2) without reverse thrust, the contribution of $V_{d\ \text{residual}}$ is small where $\gamma = 1.2\ \gamma_0$ and $\varphi = 0.6$; it increases with φ but increases where γ decreases; 3) the reverse thrust always reduces the contributions of μ and V_t; and 4) the reverse thrust introduces the parameter τ and increases the contribution of $V_{d\ \text{residual}}$. In all of the cases, the combined contributions of $V_{d\ \text{residual}}$ and τ is more than 50% at $\nu = \nu_1$.

Distance for $P = 10^{-5}$

Figure 2 shows the distances calculated for $P = 10^{-5}$, denoted $S_{10^{-5}}$, compared with S_R, which is the distance corresponding to $V_{CO\ \max}$ taken as a reference $\varphi = 1$ is taken. Both distances are plotted against ν. The slope of the curves $S_{10^{-5}}$ is smaller than the slope of the curves S_R, and it tends to become smaller still where ν is large. For $\varphi = 1$ and $\gamma = \gamma_0$, a reduction of ν of 10% results in an increase of S_R of 2% but an increase of $S_{10^{-5}}$ of only 0.5%. The effect of variations of the reverse thrust installed on various aircraft of the same series is thus negligible, where $S_{10^{-5}}$ is taken as the yardstick for comparison rather than S_R or S measured at V_{A_t}.

Since normally the reference distance is measured on the actual aircraft in specified conditions of V_t, V_d, h, τ, μ, and γ and since it can be assumed that the derivatives are proportional to the distance, it is convenient to express the results in terms of a field length factor F, defined as $S_{10^{-5}}/S_R$. Figure 2 shows F plotted against ν, φ, and γ.

Approximate Formulas

It is possible to express approximately S_P in the form

$$S = S_1 + \alpha(P, \sigma_\tau, \nu_n)S_0 \tag{5}$$

where S_0 is the distance corresponding to $V_{CO\ \max}$ without reverse thrust, S_1 is the distance corresponding to $V_{CO\ \max}$ with reverse thrust, α is a coefficient, function of the P selected, of σ_τ and ν_n, and S is the approximate value of S_P; σ_τ is indicated because of the reservation made on its assessment, and ν_n is the nominal value of the reverse thrust, taken at V_{A_t}, divided by W.

Formula (5) can be made fairly accurate. It has the advantage of containing a term ν_n easily assessed and two distances that can be measured in flight tests, landing on the reference wet runway (or corrected to the reference wet runway). It avoids measurement or assessment of γ, η (efficiency of the braking system), or β (reduction of lift because of the spoilers action). It is valid independently of operational parameters, provided that γ and φ are of the order of magnitude assumed in this study. For $P = 10^{-5}$, and $\sigma_\tau = 1$ sec, a value of α is $\alpha = 0.20 + 0.25\ \nu_n$. The value

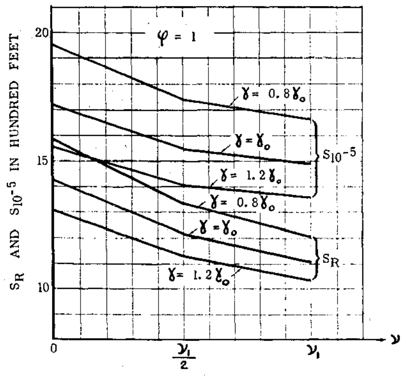

Fig. 2 Distance S_P for $P = 10^{-5}$ and reference distance S_R (measured at $V_t = V_{CO\ \max}$) vs ν.

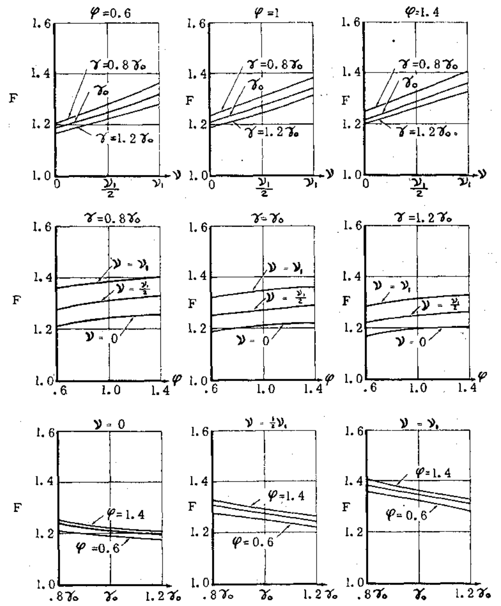

Fig. 3 Field length factor F vs ν, φ, and γ ($F = S_{10^{-5}}/S_R$).

of S, approximate value of $S_{10^{-5}}$,

$$S = S_1 + (0.20 + 0.25\ \nu_n)S_0 \tag{6}$$

is systematically conservative where $\varphi = 0.6$, but also $\sigma_t = 1$ sec may be particularly optimistic where the time to stop the aircraft without reverse thrust is relatively large. On the other hand, aircraft having poor braking mechanisms not compensated by strong spoiler action do not represent a future trend.

Formula (6) tends to be less conservative for $\nu = 0$ than for $\nu \neq 0$, but it is also for $\nu = 0$ that the contribution of V_t is the largest (Fig. 1) and that σ_s calculated by the simplified procedure is an overestimation. Formula (5) can be further simplified as

$$\left.\begin{array}{lll} S = 1.20\ S_0 & \text{where} & \nu = 0 \\ S = S_1 + 0.27\ S_0 & \text{where} & \nu \neq 0 \end{array}\right\} \tag{7}$$

It then becomes conservative at the small values of ν, but it yields the same results as (6) at $\nu = 0$ and $\nu = \nu_1$.

Conclusions

The main conclusion is that judgment of the merit of a device, which is not based on a comparison of S_P with and without the device, might be misleading. The drag introduced or increased by the device is not its only effect, since it may at the same time change the distribution of the landing distance, and a part of the advantage may be lost. On the other hand, devices that would not change the drag but that would have the effect of stabilizing one of the parameters that affect the distance may be advantageous, as reducing S_P, although they do not alter the reference distance. Figure 3 indicates that increasing the aerodynamic drag is a double gain, since it reduces the effect of the variability of other variables. Figure 3 also shows that increasing φ, by making the wheel braking devices more efficient and increasing the action of the spoilers, increases the effect of the variability of other parameters. The reference distance (distance corresponding to $V_{CO\ \max}$, for instance) may be reduced, but it will have to be multiplied by a higher field length factor. Figure 3 also shows that increasing the reverse thrust has the same effect as an increase of φ, but much more marked, because of the increasing importance of the variation of τ and V_d. Those effects are well reflected in the simplified formulas (6) and (7), since these formulas show that full credit should be taken of the drag introduced by a device only if it reduces S_0 as well as S_1.

The simplified formulas, however, should be used with caution in case of departure from the numerical assumptions made in this study. It is not evident, without further analysis, whether they are valid at small values of a which correspond to great variability of V_d. They should not be used where power is used for lift augmentation during the approach and reduced after passing the threshold, or where there are other departures from the fundamental assumptions made in the study. It is not certain, in those cases, whether the simplified procedure for combining the distribution is sufficiently precise.

These remarks show the importance of the continuation of studies like the present one. It may be objected that the existing regulations, imposing a rigid field length factor on a specified reference distance, would not permit taking credit for a device, which would reduce S_P without affecting the reference distance (like automatic application of reverse thrust, or automatic touchdown, etc.) and which would not permit taking credit for a device over and above the reduction of the reference distance that it produces. The answer is, first, that regulations are permanently in the course of evolution; and, second, that calculations of S_P may sometimes show that the field length required by the regulations, in fact, provide for a level of safety higher or lower than the level of safety generally intended. The aircraft manufacturer should be the first to be aware of this fact when it occurs.

Another very important conclusion is that, although much time and effort had to be spent on supplementing the information available in the statistics, the estimation of σ_T and of $\sigma_{h_t \text{ residual}}$ was made with reservations. It is desirable that more statistical information become available in the future, particularly if the method or the examples given in this paper would be used as a basis for airworthiness regulations.

References

[1] "Report of the standing committee on performance," International Civil Aviation Organization Doc. 7401 (August 1953).

[2] "Developments in landing distance requirements in the United Kingdom," presented to the International Civil Aviation Organization Airworthiness Committee by the member designated by U.K. AIR/C-WP/70 (July 3, 1959).

[3] "Measured landing performance of jet airplanes near threshold," presented to the International Civil Aviation Organization Airworthiness Committee by the member designated by U. S. AIR C-WP/129 (October 10, 1960).

[4] "Photographic measurements of landings at London Airport," presented to the International Civil Aviation Organization Airworthiness Committee by the member designated by U.K. AIR C-WP/163 (February 21, 1962).

[5] "Information on the speed of aeroplanes at threshold and touchdown in actual landings," presented to the International Civil Aviation Organization Airworthiness Committee by the member designated by Netherlands, AIR C-WP/165 (February 22, 1962).

[6] "Measured landing performance of jet airplanes near threshold," presented to the International Civil Aviation Organization Airworthiness Committee by the member designated by U. S. AIR C-WP/185 (April 16, 1962).

[7] "Photographic measurements of landings at Preswick Airport," presented to the International Civil Aviation Organization Airworthiness Committee by the member designated by U.K. AIR C-WP/187 (April 16, 1962).

[8] "Analysis of operational landing statistics of turbine engined aeroplanes," presented to the International Civil Aviation Organization Airworthiness Committee by the member designated by U.K. AIR C-WP/195 (May 5, 1962).

[9] Kendall, M. G., *The Advanced Theory of Statistics* (Charles Griffin and Co., Ltd., London).

[10] Holzhauser, C. A., Deckert, W. H., Quigley, H. C., and Kelly, M. W., "Design and operating considerations of commercial STOL transports," J. Aircraft **2**, 87–93 (1965).

[11] Logie, R. H., White, A. J., and Galey, D. W., "Evaluation of a combined thrust reverser-jet deflector concept," AIAA Preprint 64-287 (1964).

[12] Horne, W. B. and Leland, T. J. W., "Influence of the tread pattern and runway surface conditions on braking friction and rolling resistance of a modern aircraft," NASA TN D-1376 (September 1962).